Unit Title: Rational and Irrational Expressions; Polynomials

Stage 1: Desired Results

Standards & Indicators:

Arithmetic with Polynomials and Rational Expressions A – APR

A. Perform arithmetic operations on polynomials

1. Understand that polynomials form a system analogous to the integers, namely, they are closed under the operations of addition, subtraction, and multiplication; add, subtract, and multiply polynomials.

B. Understand the relationship between zeros and factors of polynomials

2. Know and apply the Remainder Theorem: For a polynomial p(x) and a number a, the remainder on division by x - a is p(a), so p(a) = 0 if and only if (x - a) is a factor of p(x).

3. Identify zeros of polynomials when suitable factorizations are available, and use the zeros to construct a rough graph of the function defined by the polynomial.

C. Use polynomial identities to solve problems

4. Prove polynomial identities and use them to describe numerical relationships. For example, the difference of two squares; the sum and difference of two cubes; the polynomial identity (x2 + y2) 2 = (x2 - y2) 2 + (2xy) 2 can be used to generate Pythagorean triples.

5. (+) Know and apply the Binomial Theorem for the expansion of (x + y) n in powers of x and y for a positive integer n, where x and y are any numbers, with coefficients determined for example by Pascal's Triangle.

D. Rewrite rational expressions

- 6. Rewrite simple rational expressions in different forms; write a(x)/b(x) in the form q(x) + r(x)/b(x), where a(x), b(x), q(x), and r(x) are polynomials with the degree of r(x) less than the degree of b(x), using inspection, long division, or, for the more complicated examples, a computer algebra system.
- 7. (+) Understand that rational expressions form a system analogous to the rational numbers, closed under addition, subtraction, multiplication, and division by a nonzero rational expression; add, subtract, multiply, and divide rational expressions.

Career Readiness, Life Literacies and Key Skills			
Standard	Performance Expectations	Core Ideas	
9.4.12.Cl.1	Demonstrate the ability to reflect, analyze, and use creative skills and ideas (e.g.1.1.12prof.CR3a).	With a growth mindset, failure is an important part of success.	
9.4.12.GCA.1	Collaborate with individuals to analyze a variety of potential solutions to climate change effects and determine why some solutions (e.g., political. economic, cultural) may work	Solutions to the problems faced by a global society require the contribution of individuals with different points of view and experiences.	

	better than others (e.g., SL.11-12.1., HS-ETS1-1, HS-ETS1-2, HS-ETS1-4, 6.3.12.GeoGI.1, 7.1.IH.IPERS.6, .1.IL.IPERS.7, 8.2.12.ETW.3).		
9.4.12.CT.1	Identify problem-solving strategies used in the development of an innovative product or practice (e.g., 1.1.12acc.C1b, 2.2.12.PF.3).		Collaboration with individuals with diverse experiences can aid in the problem-solving process, particularly for global issues where diverse solutions are needed.
Central Idea/Enduring Und	derstanding:	Essential/Guiding Que	estion:
Rational expressions form a system analogous to the rational numbers, closed under addition, subtraction, multiplication, and division by a nonzero rational expression.		How can we rewrite rati using inspection, long d How can we subtract, m	onal expressions in different forms ivision, or computer algebra? nultiply, and divide rational expressions?
		How can we know and a	apply the Remainder Theorem?
		How can we identify zer factorizations are availa rough graph of the funct	ros of polynomials when suitable ble, and use the zeros to construct a tion defined by a polynomial
		How can we know and a expansion of (x+ y)n in n, where x and y are an for example by Pascal's	apply the Binomial Theorem for the powers of x and y for a positive integer y numbers, with coefficients determined s Triangle?
Content:		Skills(Objectives): Simplify radical express rationalize the denomina	ions and rational exponents and to ator.
Rational Expressions Zeros of Polynomials		Reduce and perform op	erations with rational expressions.
Remainder Theorem BinomialTheorem		Determine the domain o	of rational expressions.
		Simplify compound ratio	onal expressions.
		Rewrite simple rational inspection, long division complicated examples.	expressions in different forms using , or computer algebra system for more
		Add, subtract, multiply, a	and divide rational expressions
		Apply the Remainder Th	neorem
		Identify zeros of polynor available, and use the z function defined by the	mials when suitable factorizations are eros to construct a rough graph of the polynomial.
		Perform synthetic division	on.

	Find the real and complex zeros of a function.	
	Understand and use the Fundamental Theorem of Algebra.	
	Expand binomials.	
	Know and apply the Binomial Theorem for the expansion of (x+ y)n in powers of x and y for a positive integer n, where x and y are any numbers, with coefficients determined for example by Pascal's Triangle	
Interdisciplinary Connections:	·	
Interdisciplinary connections are integrated in each u	init with connections to the mathematical practices.	
 Make sense of problems and persevere in solving them Reason abstractly and quantitatively Construct viable arguments and critique the reasoning of others Model with mathematics Use appropriate tools strategically Attend to precision Look for and make use of structure Look for and express regularity in repeated reasoning 		
Stage 2: A	Assessment Evidence	
Performance Task(s):	Other Evidence:	
A.APR.B.2 The Missing Coefficient <u>https://www.illustrativemathematics.org/conte</u> <u>nt-standards/HSA/APR/B/2/tasks/592</u>	Written and Online Assignments Exit Cards Mid Chapter Quizzes End of Chapter Assessments	
A.APR.B.3 Graphing from Factors III <u>https://www.illustrativemathematics.org/conte</u> <u>nt-standards/HSA/APR/B/3/tasks/1657</u>		
A.APR.D.6 Combined Fuel Efficiency <u>https://www.illustrativemathematics.org/conte</u> <u>nt-standards/HSA/APR/D/6/tasks/825</u>		
Stage	3: Learning Plan	
Learning Opportunities/Strategies:	Resources:	
Turn and talk	PreCalculus, Graphical, Numerical, Algebraic, Seventh Edition,	
Student driven activities	copyright 2007, Finney, Demana, Waits, Kennedy IXL Delta math	

Think, Pair, Share strategy	Edulastic
	Kahoot
Small group collaboration	Classkick
	Khan Academy
Videos/apps when appropriate	Lesson Presentations and Videos
	Graphing Calculator
	Desmos
	Google Apps for Education
	LGBT and Disabilities Resources:
	LGBTQ-Inclusive Lesson & Resources by Garden
	State Equality and Make it Better for Youth
	LGBTQ+ Books
	DEI Resources:
	Learning for Justice
	GLSEN Educator Resources
	 Supporting LGBTQIA Youth Resource List
	 Respect Ability: Fighting Stigmas, Advancing
	Opportunities
	NJDOE Diversity, Equity & Inclusion Educational
	Resources
	Diversity Calendar

Differentiation

*Please note: Teachers who have students with 504 plans that require curricular accommodations are to refer to Struggling and/or Special Needs Section for differentiation

High-Achieving	On Grade Level	Struggling Students	Special Needs/ELL
Students	Students		
Khan Academy Project based learning Tablets	Tutoring Tables Graphic organizers	Provide a highly structured, predictable learning environment	Any student requiring further accommodations and/or modifications will have them individually listed in
Challenging problems with higher degree of difficulty Higher order thinking questions Differentiation of pacing and activities Differentiation of learning	Differentiation of learning strategies: visual, auditory, kinetic and cooperative Technology connection Practice Assignments	Provide organizers/study guides Lessons designed to the style of learning that matches the student Cooperative Learning	their 504 Plan or IEP. These might include, but are not limited to: breaking assignments into smaller tasks, giving directions through several channels (auditory, visual, kinesthetic, model), and/or small group instruction for reading/writing
strategies: visual, auditory, kinetic and cooperative Enrichment and extension Technology connection Practice assignments Puzzle time activities Record and practice journal	Puzzle time activities Record and practice journal Differentiating the lesson activities Lesson tutorials Skills review handbook	Positive reinforcement Announce test with adequate prep time Lessons presentation available on google classroom Frequent check for understanding Break down task into manageable units One-on-one instruction Tutoring	ELL supports should include, but are not limited to, the following:: Extended time Provide visual aids Repeated directions Differentiate based on proficiency Provide word banks Allow for translators, dictionaries

Pair student with a high achieving	
student	

Unit Title: Functions
Stage 1: Desired Results
Standards & Indicators:
 Interpreting Functions F-IF A. Understand the concept of a function and use function notation 1. Understand that a function from one set (called the domain) to another set (called the range) assigns to each element of the domain exactly one element of the range. If <i>f</i> is a function and <i>x</i> is an element of its domain, then <i>f</i>(<i>x</i>) denotes the output of <i>f</i> corresponding to the input <i>x</i>. The graph of <i>f</i> is the graph of the equation <i>y</i> = <i>f</i>(<i>x</i>). 2. Use function notation, evaluate functions for inputs in their domains, and interpret statements that use function notation in terms of a context.
 3. Recognize that sequences are functions, sometimes defined recursively, whose domain is a subset of the integers. For example, the Fibonacci sequence is defined recursively by f(0) = f(1) = 1, f(n+1) = f(n) + f(n-1) for n ≥ 1.
 B. Interpret functions that arise in applications in terms of the context 4. For a function that models a relationship between two quantities, interpret key features of graphs and tables in terms of the quantities, and sketch graphs showing key features given a verbal description of the relationship. Key features include: intercepts; intervals where the function is increasing, decreasing, positive, or negative; relative maximums and minimums; symmetries; end behavior; and periodicity.*
 5. Relate the domain of a function to its graph and, where applicable, to the quantitative relationship it describes. For example, if the function h(n) gives the number of person-hours it takes to assemble n engines in a factory, then the positive integers would be an appropriate domain for the function.* 6. Calculate and interpret the average rate of change of a function (presented symbolically or as a table) over a specified interval. Estimate the rate of change from a graph.*
C. Analyze functions using different representations
 7. Graph functions expressed symbolically and show key features of the graph, by hand in simple cases and using technology for more complicated cases.* a. Graph linear and quadratic functions and show intercepts, maxima, and minima. b. Graph square root, cube root, and piecewise-defined functions, including step functions and absolute value
functions. c. Graph polynomial functions, identifying zeros when suitable factorizations are available, and showing end behavior.
d. (+) Graph rational functions, identifying zeros and asymptotes when suitable factorizations are available, and showing end behavior.
e. Graph exponential and logarithmic functions, showing intercepts and end behavior, and trigonometric functions, showing period, midline, and amplitude.

- 8. Write a function defined by an expression in different but equivalent forms to reveal and explain different properties of the function.
 - a. Use the process of factoring and completing the square in a quadratic function to show zeros, extreme values, and symmetry of the graph, and interpret these in terms of a context.
- 9. Compare properties of two functions each represented in a different way (algebraically, graphically, numerically in tables, or by verbal descriptions). *For example, given a graph of one quadratic function and an algebraic expression for another, say which has the larger maximum.*

Building Functions F-BF

A. Build a function that models a relationship between two quantities

- 1. Write a function that describes a relationship between two quantities.*
 - a. Determine an explicit expression, a recursive process, or steps for calculation from a context.
 - b. Combine standard function types using arithmetic operations. *For example, build a function that models the temperature of a cooling body by adding a constant function to a decaying exponential, and relate these functions to the model.*
 - c. (+) Compose functions. For example, if T(y) is the temperature in the atmosphere as a function of height, and h(t) is the height of a weather balloon as a function of time, then T(h(t)) is the temperature at the location of the weather balloon as a function of time.
- 2. Write arithmetic and geometric sequences both recursively and with an explicit formula, use them to model situations, and translate between the two forms.*

B. Build new functions from existing functions

- 3. Identify the effect on the graph of replacing f(x) by f(x) + k, k f(x), f(kx), and f(x + k) for specific values of k (both positive and negative); find the value of k given the graphs. Experiment with cases and illustrate an explanation of the effects on the graph using technology. *Include recognizing even and odd functions from their graphs and algebraic expressions for them.*
- 4. Find inverse functions. a. Solve an equation of the form f(x) = c for a simple function f that has an inverse and write an expression for the inverse. For example, f(x) =2 x3 or f(x) = (x+1)/(x-1) for x ≠1.
 b. (+) Verify by composition that one function is the inverse of another. c. (+) Read values of an inverse function from a graph or a table, given that the function has an inverse. d. (+) Produce an invertible function from a non-invertible function by restricting the domain.

Career Readiness, Life Literacies and Key Skills			
Standard	Performance Expectations	Core Ideas	
9.4.12.Cl.1	Demonstrate the ability to reflect, analyze, and use creative skills and ideas (e.g., 1.1.12prof.CR3a).	With a growth mindset, failure is an important part of success.	
9.4.12.CT.1	Identify problem-solving strategies used in the development of an innovative product or practice (e.g., 1.1.12acc.C1b, 2.2.12.PF.3).	Collaboration with individuals with diverse experiences can aid in the problem-solving process, particularly for global issues where diverse solutions are needed.	

5. (+) Use the inverse relationship between exponents and logarithms to solve problems involving logarithms and exponents.

9.4.12.GCA.1	Collaborate with individuals to analyze a variety of potential solutions to climate change effects and determine why some solutions (e.g., political. economic, cultural) may work better than others (e.g., SL.11-12.1., HS-ETS1-1, HS-ETS1-2, HS-ETS1-4, 6.3.12.GeoGl.1, 7.1.IH.IPERS.6, 7.1.IL.IPERS.7, 8.2.12.ETW.3).		Solutions to the problems faced by a global society require the contribution of individuals with different points of view and experiences.
Central Idea/Enduring Und	derstanding:	Essential/Guiding Que	estion:
Linear relations and functions have straight line graphs. The rate of change of a linear function is known as the slope and can be found using any two points on the line. The equation of a line can be written whenever two points or a point and the slope of the line are known. A line of fit can be used to approximate the relation between domain and range values of a data set that exhibits a linear trend		How can we understand domain) to another set (element of the domain exactly one element of t How can we show if f is domain, then f(x) denote x?	d that a function from one set (called the (called the range) assigns to each the range? a function and x is an element of its es the output corresponding to the input
The inverse of a function ca exchanging the domain and Functions with a variable un are called radical functions. functions are square root fu functions. When solving rad isolate the radical, then rais power equal to the index of solve the resulting equation	n be found by range of the function. Ider a radical symbol Two types of radical nctions and cube root ical equations, first e each side to the the radical, and finally,	How can we show that the quation y = f(x)? How can we compose fit between two quantities? How can we solve an experimental function f that has an invitation of that has an invitative of another? How can we verify by continuerse of another? How can we read values given that the function here and the function here and the function here are non-invertible function be. How do we graph linear show intercepts, maximal there are an are graph expensioned and the function are showing intercepts and the function of the functi	the graph off is the graph of the unctions that model a relationship quation of the form f(x) = c for a simple verse and write an expression for the omposition that one function is the s of a function from a table or a graph, has an inverse? In invertible function from a by restricting the domain? It and quadratic functions and a, and minima? It and quadratic functions and behavior? It and logarithmic functions, end behavior? Inometric functions, showing period, It the inverse relationship between ns?

	How can we use this relationship to solve problems involving logarithms and exponents?
Content:	Skills(Objectives):
Midpoint and Distance Formula	Determine domain and range of a function.
Compositions of Functions Inverse Functions Maxima and Minima	Determine if a function is increasing, decreasing or constant.
Polynomial Functions	Determine if a function is odd or even.
	Find combinations and compositions of functions.
	Find and verify the inverse of a function.
	Determine the parts of the Cartesian plane.
	Use the midpoint and distance formulas.
	Find x and y intercepts.
	Find maximums and minimums•
	Test for symmetry
	Formulate the equation of a circle.
	Formulate equations of lines and lines parallel and perpendicular to a line.
	Compose functions that model a relationship between two quantities.
	Solve an equation of the form $f(x) = c$ for a simple function f that has an inverse and write an expression for the inverse.
	Verify by composition that one function is the inverse of another.
	Read values of an inverse function from a graph or a table, given that the function has an inverse.
	Produce an invertible function from a non-invertible function by restricting the domain.
	Find the real and complex zeros of a function.
	Formulate the standard form of and graph quadratic equations. Graph using translations of functions.
	Analyze and compare graphs of functions of higher degree polynomials.

	Graph exponential functions and determine their behaviors.
	Compute a logarithm value of a number with and without the aid of a calculator.
	Graph logarithmic functions and determine their behaviors. Explore the properties of the natural exponential function and the natural logarithm.
	Expand and simplify expressions using the properties of logarithms.
	Solve exponential and logarithmic equations.
	Solve real-life problems involving exponential and logarithmic equations
	Understand the inverse relationship between exponents and logarithms and use this relationship to solve problems involving logarithms and exponents.
	Solve exponential and logarithmic equations.
	Solve real-life problems involving exponential and logarithmic equations.
Interdisciplinary Connections:	
Interdisciplinary connections are integrated in each u	init with connections to the mathematical practices.

- 1. Make sense of problems and persevere in solving them
- 2. Reason abstractly and quantitatively
- 3. Construct viable arguments and critique the reasoning of others
- 4. Model with mathematics
- 5. Use appropriate tools strategically
- 6. Attend to precision
- 7. Look for and make use of structure
- 8. Look for and express regularity in repeated reasoning

Stage 2: Assessment Evidence		
Performance Task(s):	Other Evidence: Written and Online Assignments	
F.BF.A.1b A Sum of Functions https://www.illustrativemathematics.org/conte nt-standards/HSF/BF/A/1/tasks/230	Exit Cards Mid Chapter Quizzes End of Chapter Assessments End of Unit Common Assessments	
F.BF.B.4a		

Temperatures in degrees Fahrenheit and		
Celsius		
https://www.illustrativemathematics.org/conte		
nt-standards/HSF/BF/B/4/tasks/501		
Stage	3: Learning Plan	
Learning Opportunities/Strategies:	Resources:	
Turn and talk	PreCalculus, Graphical, Numerical, Algebraic, Seventh Edition, copyright 2007, Finney, Demana, Waits, Kennedy IXI	
Student driven activities	Delta math Edulastic	
Think, Pair, Share strategy	Kahoot	
Small group collaboration	Khan Academy	
Videos/apps when appropriate	Graphing Calculator Desmos Google Apps for Education	
	LGBT and Disabilities Resources: <u>LGBTQ-Inclusive Lesson & Resources by Garden State Equality and Make it Better for Youth</u> <u>LGBTQ+ Books</u> 	
	 DEI Resources: <u>Learning for Justice</u> <u>GLSEN Educator Resources</u> <u>Supporting LGBTQIA Youth Resource List</u> <u>Respect Ability: Fighting Stigmas, Advancing Opportunities</u> <u>NJDOE Diversity, Equity & Inclusion Educational Resources</u> <u>Diversity Calendar</u> 	

Differentiation *Please note: Teachers who have students with 504 plans that require curricular accommodations are to refer to Struggling and/or Special Needs Section for differentiation

High-Achieving	On Grade Level	Struggling Students	Special Needs/ELL
Students	Students		
Khan Academy	Tutoring	Provide a highly	Any student requiring further
Project based learning	Tables	structured, predictable	accommodations and/or modifications
Tablets	Graphic organizers	learning environment	will have them individually listed in
Challenging problems	Differentiation of	Provide	their 504 Plan or IEP. These might
with higher degree of	learning strategies:	organizers/study	include, but are not limited to:
difficulty	visual, auditory,	guides	breaking assignments into smaller
Higher order thinking	kinetic and	Lessons designed to	tasks, giving directions through
questions	cooperative	the style of learning	several channels (auditory, visual,
Differentiation of pacing	Technology	that matches the	kinesthetic, model), and/or small
and activities	connection	student	group instruction for reading/writing
Differentiation of learning	Practice Assignments	Cooperative Learning	
strategies: visual,	Puzzle time activities	Positive reinforcement	ELL supports should include, but are
auditory, kinetic and	Record and practice		not limited to, the following::

			—
cooperative	journal	Announce test with	Extended time
Enrichment and extension	Differentiating the	adequate prep time	Provide visual aids
Technology connection	lesson activities	Lessons presentation	Repeated directions
Practice assignments	Lesson tutorials	available on google	Differentiate based on proficiency
Puzzle time activities	Skills review	classroom	Provide word banks
Record and practice journal	handbook	Frequent check for understanding Break down task into manageable units One-on-one instruction Tutoring Pair student with a high achieving student	Allow for translators, dictionaries

Unit Title: The Complex Number System

Stage 1: Desired Results

Standards & Indicators:

The Complex Number System N -CN

A. Perform arithmetic operations with complex numbers.

1. Know there is a complex number i such that $i^2 = -1$, and every complex number has the form a + bi with a and b real.

2. Use the relation $i^2 = -1$ and the commutative, associative, and distributive properties to add, subtract, and multiply complex numbers.

3. (+) Find the conjugate of a complex number; use conjugates to find moduli and quotients of complex numbers.

B. Represent complex numbers and their operations on the complex plane.

4. (+) Represent complex numbers on the complex plane in rectangular and polar form (including real and imaginary numbers), and explain why the rectangular and polar forms of a given complex number represent the same number.

5. (+) Represent addition, subtraction, multiplication, and conjugation of complex numbers geometrically on the complex plane; use properties of this representation for computation. For example, $(-1 + \sqrt{3}i)3 = 8$ because $(-1 + \sqrt{3}i)$ has modulus 2 and argument 120°.

6. (+) Calculate the distance between numbers in the complex plane as the modulus of the difference, and the midpoint of a segment as the average of the numbers at its endpoints.

C. Use complex numbers in polynomial identities and equations.

7. Solve quadratic equations with real coefficients that have complex solutions.

8. (+) Extend polynomial identities to the complex numbers. For example, rewrite $x^2 + 4$ as (x + 2i)(x - 2i).

9. (+) Know the Fundamental Theorem of Algebra; show that it is true for quadratic polynomials.

Career Readiness, Life Literacies and Key Skills			
Standard	Performance Expectations		Core Ideas
9.4.12.Cl.1	Demonstrate the ability to reflect, analyze, and use creative skills and ideas (e.g., 1.1.12prof.CR3a).		With a growth mindset, failure is an important part of success.
9.4.12.CT.1	Identify problem-solving strategies used in the development of an innovative product or practice (e.g., 1.1.12acc.C1b, 2.2.12.PF.3).		Collaboration with individuals with diverse experiences can aid in the problem-solving process, particularly for global issues where diverse solutions are needed.
9.4.12.GCA.1	Collaborate with individ variety of potential solut effects and determine w (e.g., political. economic better than others (e.g., HS-ETS1-1, HS-ETS1- 6.3.12.GeoGI.1, 7.1.IH. 7.1.IL.IPERS.7, 8.2.12.	uals to analyze a tions to climate change vhy some solutions c, cultural) may work , SL.11-12.1., 2, HS-ETS1-4, IPERS.6, ETW.3).	Solutions to the problems faced by a global society require the contribution of individuals with different points of view and experiences.
Central Idea/Enduring Un	derstanding:	Essential/Guiding Que	estion:
Complex numbers are num parts ; a real number and an Complex numbers are the b intricate math, such as alge applied to many aspects of electronics and electromage	bers that consist of two n imaginary number. building blocks of more bra. They can be real life, especially in netism.	How can we solve quad that have complex solut How can we extend pol numbers?	Iratic equations with real coefficients tions? ynomial identities to the complex
		How can we learn the F show that it is true for g	undamental Theorem of Algebra and uadratic polynomials?
Content: Quadratic equations Polynomial Identities Fundamental Theorem of A Complex Numbers	lgebra	Skills(Objectives): Solve quadratic equatio complex solutions. Extend polynomial ider example, rewrite x2 + 4	Theorem of Alexberry discretion in the set of the set
		Know the Fundamental true for quadratic polyn	Theorem of Algebra; show that it is omials.
Interdisciplinary Connect	ions:	1 1 1	

Interdisciplinary connections are integrated in each unit with connections to the mathematical practices.

 Make sense of problems and persevere in solving them Reason abstractly and quantitatively Construct viable arguments and critique the reasoning of others Model with mathematics Use appropriate tools strategically Attend to precision 		
7. Look for and make use of structure		
8. Look for and express regularity in repeated reason	ling	
Stage 2: A	Assessment Evidence	
Performance Task(s): N.CN.7 Completing the square http://tasks.illustrativemathematics.org/content-sta ndards/HSN/CN/C/7/tasks/1690	Other Evidence: Written and Online Assignments Exit Cards Mid Chapter Quizzes End of Chapter Assessments End of Unit Common Assessments	
Store	2: Learning Blan	
Stage	3: Learning Plan	
Turn and talk	Resources: PreCalculus, Graphical, Numerical, Algebraic, Seventh Edition, copyright 2007, Finney, Demana, Waits, Kennedy	
Student driven activities	Delta math	
Think, Pair, Share strategy	Edulastic Kahoot Classkick	
Small group collaboration	Khan Academy	
Videos/apps when appropriate	Lesson Presentations and Videos Graphing Calculator Desmos Google Apps for Education	
	LGBT and Disabilities Resources: • LGBTQ-Inclusive Lesson & Resources by Garden State Equality and Make it Better for Youth • LGBTQ+ Books	
	 DEI Resources: Learning for Justice GLSEN Educator Resources Supporting LGBTQIA Youth Resource List Respect Ability: Fighting Stigmas, Advancing Opportunities NJDOE Diversity, Equity & Inclusion Educational Resources Diversity Calendar 	

Differentiation *Please note: Teachers who have students with 504 plans that require curricular accommodations are to refer to Struggling and/or Special Needs Section for differentiation

High-Achieving	On Grade Level	Struggling Students	Special Needs/ELL
Khan Academy Project based learning Tablets Challenging problems with higher degree of difficulty Higher order thinking questions Differentiation of pacing and activities Differentiation of learning strategies: visual, auditory, kinetic and cooperative Enrichment and extension Technology connection Practice assignments Puzzle time activities Record and practice journal	Tutoring Tables Graphic organizers Differentiation of learning strategies: visual, auditory, kinetic and cooperative Technology connection Practice Assignments Puzzle time activities Record and practice journal Differentiating the lesson activities Lesson tutorials Skills review handbook	Provide a highly structured, predictable learning environment Provide organizers/study guides Lessons designed to the style of learning that matches the student Cooperative Learning Positive reinforcement Announce test with adequate prep time Lessons presentation available on google classroom Frequent check for understanding Break down task into manageable units One-on-one instruction Tutoring Pair student with a high achieving student	Any student requiring further accommodations and/or modifications will have them individually listed in their 504 Plan or IEP. These might include, but are not limited to: breaking assignments into smaller tasks, giving directions through several channels (auditory, visual, kinesthetic, model), and/or small group instruction for reading/writing ELL supports should include, but are not limited to, the following:: Extended time Provide visual aids Repeated directions Differentiate based on proficiency Provide word banks Allow for translators, dictionaries

Unit Title: Trigonometric Functions, Similarity, Right Triangles, and Trigonometry

Stage 1: Desired Results

Standards & Indicators:

Trigonometric Functions F-TF

A. Extend the domain of trigonometric functions using the unit circle

- 1. Understand radian measure of an angle as the length of the arc on the unit circle subtended by the angle.
- 2. Explain how the unit circle in the coordinate plane enables the extension of trigonometric functions to all real numbers, interpreted as radian measures of angles traversed counterclockwise around the unit circle.
- 3. (+) Use special triangles to determine geometrically the values of sine, cosine, tangent for $\pi/3$, $\pi/4$ and $\pi/6$, and use the unit circle to express the values of sine, cosines, and tangent for $\pi-x$, $\pi+x$, and $2\pi-x$ in terms of their values for x, where x is any real number.

4. (+) Use the unit circle to explain symmetry (odd and even) and periodicity of trigonometric functions.

B. Model periodic phenomena with trigonometric functions

- 5. Choose trigonometric functions to model periodic phenomena with specified amplitude, frequency, and midline.★
- 6. (+) Understand that restricting a trigonometric function to a domain on which it is always increasing or always decreasing allows its inverse to be constructed.
- 7. (+) Use inverse functions to solve trigonometric equations that arise in modeling contexts; evaluate the solutions using technology, and interpret them in terms of the context.★

C. Prove and apply trigonometric identities

- 8. Prove the Pythagorean identity $\sin 2(\theta) + \cos 2(\theta) = 1$ and use it to find $\sin(\theta)$, $\cos(\theta)$, or $\tan(\theta)$ given $\sin(\theta)$, $\cos(\theta)$, or $\tan(\theta)$ and the quadrant of the angle.
- 9. (+) Prove the addition and subtraction formulas for sine, cosine, and tangent and use them to solve problems.

Similarity, Right Triangles, and Trigonometry G-SRT

C. Define trigonometric ratios and solve problems involving right triangles

6. Understand that by similarity, side ratios in right triangles are properties of the angles in the triangle, leading to definitions of trigonometric ratios for acute angles.

7. Explain and use the relationship between the sine and cosine of complementary angles.

8. Use trigonometric ratios and the Pythagorean Theorem to solve right triangles in applied problems.

D. Apply trigonometry to general triangles

9. (+) Derive the formula A = 1/2 ab sin(C) for the area of a triangle by drawing an auxiliary line from a vertex perpendicular to the opposite side.

10. (+) Prove the Laws of Sines and Cosines and use them to solve problems.

11. (+) Understand and apply the Law of Sines and the Law of Cosines to find unknown measurements in right and non-right triangles (e.g., surveying problems, resultant forces).

Caleer Readiness, Lie Literacies and Rey Skins		
Standard	Performance Expectations	Core Ideas
9.4.12.Cl.1	Demonstrate the ability to reflect, analyze, and use creative skills and ideas (e.g., 1.1.12prof.CR3a).	With a growth mindset, failure is an important part of success.
9.4.12.CT.1	Identify problem-solving strategies used in the development of an innovative product or practice (e.g., 1.1.12acc.C1b, 2.2.12.PF.3).	Collaboration with individuals with diverse experiences can aid in the problem-solving process, particularly for global issues where diverse solutions are needed.
9.4.12.GCA.1	Collaborate with individuals to analyze a variety of potential solutions to climate change effects and determine why some solutions (e.g., political. economic, cultural) may work better than others (e.g., SL.11-12.1., HS-ETS1-1, HS-ETS1-2, HS-ETS1-4,	Solutions to the problems faced by a global society require the contribution of individuals with different points of view and experiences.

6.3.12.GeoGI.1, 7.1.IH. 7.1.IL.IPERS.7, 8.2.12	IPERS.6, ETW.3).	
Central Idea/Enduring Understanding:	Essential/Guiding Question:	
 Trigonometry is a branch of mathematics that studies relationships between the sides and angles of triangles. Trigonometry is found all throughout geometry, as every straight-sided shape may be broken into as a collection of triangles. Trigonometric functions are a way to relate the lengths of the three sides of a right triangle to the interior angles of the triangle. 	How can we understand the radian measure of an angle as the length of the arc on the unit circle subtended by the angle? How can we explain how the unit circle in the coordinate plane enables the extension of trigonometric functions to all real numbers, interpreted as radian measures of angles traversed counterclockwise around the unit circle? How can we use special triangles to determine geometrically the values of sine, cosine, tangent for n/3, n/4 and n/6, and use the unit circle to express the values of sine, cosine, and tangent for n-x, n+x, and 2n-x in terms of their values for x, where x is any real number? How can we use the unit circle to explain symmetry (odd and even) and periodicity of trigonometric functions?	
	 How can we use inverse functions to solve trigonometric equations that arise in modeling contexts; evaluate the solutions using technology, and interpret them in terms of the context? How can we simplify expressions involving trigonometric inverse functions? How can we determine reference angles? How can we understand that by similarity, side ratios in right triangles are properties of the angles in the triangle, leading to 	
	 definitions of trigonometric ratios for acute angles? How can we explain and use the relationship between the sine and cosine of complementary angles? How can we prove the Pythagorean identity sin2(8) + cos2(8) = 1 and use it to find sin(8), cos(8), or tan(8) given sin(8), cos(8), or tan(8) and the quadrant of the angle? How can we prove the addition and subtraction formulas for sine, cosine, and tangent and use them to solve problems? 	
Content:	Skills(Objectives):	
Radian and Degree measures Trigonometric functions and equations Arc Length, Radius and Central Angle Similarity of triangles	Convert from radian to degree measure and vice versa. Draw angles with radian and degree measures.	
Sine, Cosine, Tangent	Find arc length, radius and central angle (theta).	

	Use the unit circle to find trigonometric values of angles.
	Solve real-life problems using the trigonometric functions.
	Find the value of any angle using the trigonometric functions
	Use the calculator to find arcsines, arccosines, and arctangents.
	Simplify expressions involving trigonometric inverse functions.
	Determine reference angles
	Use inverse functions to solve trigonometric equations that arise in modeling contexts; evaluate the solutions using technology, and interpret them in terms of the context.
	Understand that by similarity, side ratios in right triangles are properties of the angles of the triangles.
	Define and solve problems involving right triangle
	Find two angles given a specific trigonometric value.
	Graph the sine, cosine and tangent function.
	Explain and use the relationship between the sine and cosine of complementary angles. Understand and apply definitions of trigonometric ratios for acute angles.
	Use trigonometric identities to simplify, factor, multiply, add and subtract trigonometric expressions.
	Use trigonometric identities to verify trigonometric equations.
	Solve trigonometric equations.
Interdisciplinary Connections: Interdisciplinary connections are integrated in each u	nit with connections to the mathematical practices.
1. Make sense of problems and persevere in solving	them

- 2. Reason abstractly and quantitatively
- 3. Construct viable arguments and critique the reasoning of others
- 4. Model with mathematics
- 5. Use appropriate tools strategically
- 6. Attend to precision
- 7. Look for and make use of structure
- 8. Look for and express regularity in repeated reasoning

Stage 2: Assessment Evidence			
Performance Task(s):F.TF.C.9Sum and Difference angle formulashttp://tasks.illustrativemathematics.org/content-standards/HSF/TF/C/9/tasks/1116G.SRT.C.6Defining Trigonometric Ratioshttp://tasks.illustrativemathematics.org/content-standards/HSG/SRT/C/6/tasks/1635	Other Evidence: Written and Online Assignments Exit Cards Mid Chapter Quizzes End of Chapter Assessments End of Unit Common Assessments		
Stage	3: Learning Plan		
Learning Opportunities/Strategies: Turn and talk Student driven activities Think, Pair, Share strategy Small group collaboration Videos/apps when appropriate	Resources: PreCalculus, Graphical, Numerical, Algebraic, Seventh Edition, copyright 2007, Finney, Demana, Waits, Kennedy IXL Delta math Edulastic Kahoot Classkick Khan Academy Lesson Presentations and Videos Graphing Calculator Desmos Google Apps for Education LGBT and Disabilities Resources: LGBTQ-Inclusive Lesson & Resources by Garden State Equality and Make it Better for Youth LGBTQ+ Books DEI Resources: GLSEN Educator Resources Supporting for Justice GLSEN Educator Resources Supporting LGBTQIA Youth Resource List Respect Ability: Fighting Stigmas, Advancing Opportunities NJDOE Diversity, Equity & Inclusion Educational Resources NJDOE Diversity, Equity & Inclusion Educational Resources Diversity Calendar		
Differentiation			

*Please note: Teachers who have students with 504 plans that require curricular accommodations are to refer to Struggling and/or Special Needs Section for differentiation

High-Achieving	On Grade Level	Struggling Students	Special Needs/ELL
Students	Students		
Students Khan Academy Project based learning Tablets Challenging problems with higher degree of difficulty Higher order thinking questions Differentiation of pacing and activities Differentiation of learning strategies: visual, auditory, kinetic and cooperative Enrichment and extension Technology connection Practice assignments Puzzle time activities Record and practice journal	StudentsTutoring TablesGraphic organizersDifferentiation of learning strategies: visual, auditory, kinetic and cooperative Technology connectionTechnology connectionPractice Assignments Puzzle time activities Record and practice journalDifferentiating the lesson activities Lesson tutorials Skills review handbook	Provide a highly structured, predictable learning environment Provide organizers/study guides Lessons designed to the style of learning that matches the student Cooperative Learning Positive reinforcement Announce test with adequate prep time Lessons presentation available on google classroom Frequent check for understanding Break down task into manageable units One-on-one instruction Tutoring Pair student with a high achieving student	Any student requiring further accommodations and/or modifications will have them individually listed in their 504 Plan or IEP. These might include, but are not limited to: breaking assignments into smaller tasks, giving directions through several channels (auditory, visual, kinesthetic, model), and/or small group instruction for reading/writing ELL supports should include, but are not limited to, the following:: Extended time Provide visual aids Repeated directions Differentiate based on proficiency Provide word banks Allow for translators, dictionaries

Pacing Guide			
Pre Calculus Honors	PreCalculus, Graphical, Numerical, Algebraic, Seventh Edition,	Standards	
UNIT 1 Rational and Irrational Expressions; Polynomials (22 Days)	CHAPTERS 1: (11 Days) 2: (11 Days)	A.APR.A.1 A.APR.B.2 A.APR.B.3 A.APR.C.4	
		A.APR.C.5 A.APR.D.6 A.APR.D.7	
	CHADTERS		
UNIT 2 Functions (22 Days)	CHAPTERS 3: (10 Days) 4: (12 Days)	F.IF.A.1 F.IF.A.2 F.IF.A.3 F.IF.B.4 F.IF.B.5 F.IF.B.6 F.IF.C.7 F.IF.C.8 F.IF.C.9 F.BF.A.1 F.BF.A.2 F.BF.B.3 F.BF.B.3 F.BF.B.4 F.BF.B.5	
UNIT 3 The Complex Number System (10 Days)	CHAPTERS 5: (10 Days)	N.CN.A.1 N.CN.A.2 N.CN.A.3 N.CN.B.4 N.CN.B.5 N.CN.B.6 N.CN.C.7 N.CN.C.8 N.CN.C.9	

UNIT 4	CHAPTERS	F.TF.A.1
Trigonometric Functions,	6: (16 Days)	F.TF.A.2
Similarity, Right Triangles,	7: (16 Days)	F.TF.A.3
and Trigonometry		F.TF.A.4
(32 Days)		F.TF.B.5
		F.TF.B.6
		F.TF.B.7
		F.TF.C.8
		F.TF.C.9